

INTRODUCTION

An important step in analyzing and designing transistor circuits is to represent the device with a suitable equivalent circuit. For small signal AC analysis it is common practice to consider the transistor a 4 terminal "black box" with the voltages and currents at the input and output terminals related by a set of 4 parameters. The most useful sets of such parameters to date have been the h , y , and z parameters which are described in standard texts.

There are 4 h , y , and z parameters for each of the three possible device configurations -- common emitter, common collector, and common base, making a total of 36 parameters. Some of the h , y , and z

parameters for a given transistor type are normally specified by the manufacturer, depending on the intended applications.

At frequencies above 100 MHz, h , y , and z parameters become increasingly difficult to measure because it is difficult to obtain good short and open circuits and because short circuits frequently cause the device to oscillate. However, it is possible to represent a transistor with a slightly different set of parameters, the "s" or scattering parameters, which can easily be measured up to 1 GHz with the 8405A Vector Voltmeter and which can be directly applied to high frequency circuit design. This Application Note will describe the technique for measuring s parameters and will illustrate some simple applications.

SECTION I

DEFINITION & ADVANTAGES OF S PARAMETERS

Figure 1(a) illustrates the transistor configuration used when defining s parameters. Common emitter connection is shown and similar parameters exist for common base or collector.

Figure 1(b) represents the scattering parameters for the transistor using a signal flow graph*. E_{i1} and E_{r1} are the incident and reflected voltages at the input port while E_{i2} and E_{r2} are the voltages at the output port. The equations for the network are as follows:

$$E_{r1} = s_{11} E_{i1} + s_{12} E_{i2}$$

$$E_{r2} = s_{21} E_{i1} + s_{22} E_{i2}$$

$$s_{11} = \frac{E_{r1}}{E_{i1}} \quad E_{i2} = 0 \quad \text{Achieved by connecting generator to port 1 and matching port 2 as shown in Figure 1(a)}$$

$$s_{21} = \frac{E_{r2}}{E_{i1}} \quad E_{i2} = 0$$

$$s_{22} = \frac{E_{r2}}{E_{i2}} \quad E_{i1} = 0 \quad \text{Achieved by connecting generator to port 2 and matching port 1.}$$

$$s_{12} = \frac{E_{r1}}{E_{i2}} \quad E_{i1} = 0$$

s_{11} is simply the reflection coefficient looking into port 1 with port 2 matched while s_{22} is the reflection coefficient of port 2 with port 1 matched. s_{21} and s_{12} are transmissions parameters which represent gain or loss. In all cases s is a complex number, having both magnitude and phase.

In keeping with the conventions used with h , y , and z parameters, s parameters are labeled with letter subscripts as shown in Figure 1(c).

*Kuhn, Nicholas, "Simplified Flow Graph Analysis," The Microwave Journal, November 1963, pp. 59-66.

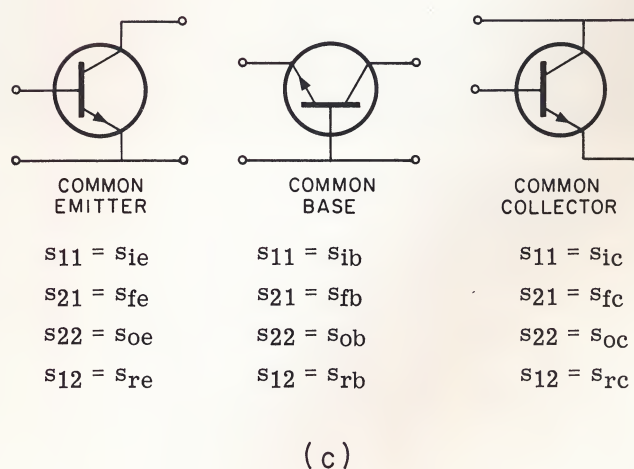
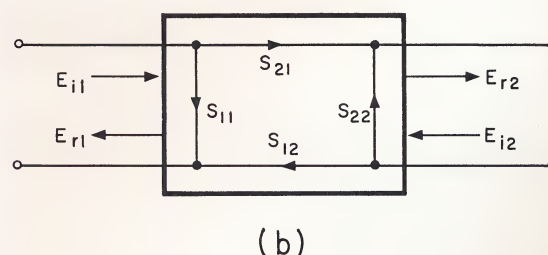
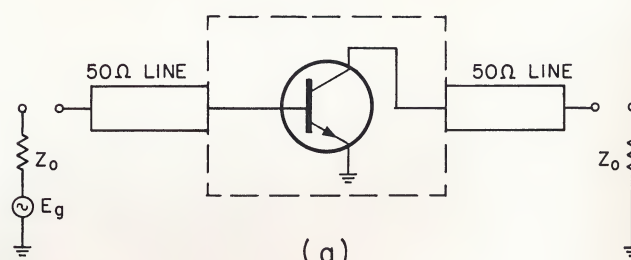


Figure 1. Two port representation of a transistor. Figure 1(a) shows a schematic picture of an NPN transistor in common emitter configuration. Figure 1(b) shows the equivalent 2 port scattering diagram. Figure 1(c) illustrates s parameter subscript notation.

Advantages

The measurement of h , y , or z parameters requires the input and output of the device to be successively open circuited and short circuited. This is difficult to do at high frequencies and typically requires a separate adjustment of tuning stubs for each frequency. Furthermore, shorting the input or output may cause the transistor to oscillate, making the measurement very difficult. With the 8405A, the measurement of s parameters is very simple over a wide frequency range after initial setup. Furthermore, since the parameters are measured with a 50 ohm load, there is much less chance for oscillations to occur. An additional advantage is that since s parameters are in units of reflection coefficient, they can be plotted directly on the Smith Chart and easily manipulated to establish optimum gain with matching networks. If h , y , or z parameters are required for a particular application, they can be obtained directly from the measured s parameters with appropriate conversion formulas. Similar conversions can be made to translate h , y , and z parameters into s parameters. The conversion formulas are given in Appendix I. Because of the ease of measurement, it may often be simpler to measure s parameters even though h , y , and z parameters are the desired end result.

SECTION II

S PARAMETER MEASUREMENT TECHNIQUE

A block diagram of a typical s parameter measurement setup is illustrated in Figure 2. The basic procedure for measuring s_{11} and s_{22} is to measure the amplitude ratio and phase angle between the incident and reflected voltages with probe B in the B_1 position. s_{11} is measured with the jig in the forward direction and s_{22} with the jig reversed. s_{12} and s_{21} , the transmitted signals, are measured with the B probe in the B_2 position with s_{21} resulting with the jig reversed.

Initial Calibration

For all s parameter measurements, it is important to set the electrical length of the signal path from the input of the first coupler to the A probe exactly the same as that to the B probe. This insures 0° phase shift between the two probes and eliminates the need for recalibration as frequency is changed. With probe B in the B_1 position this is accomplished with a shorted section and with probe B in the B_2 position with a through section. The short and through sections are

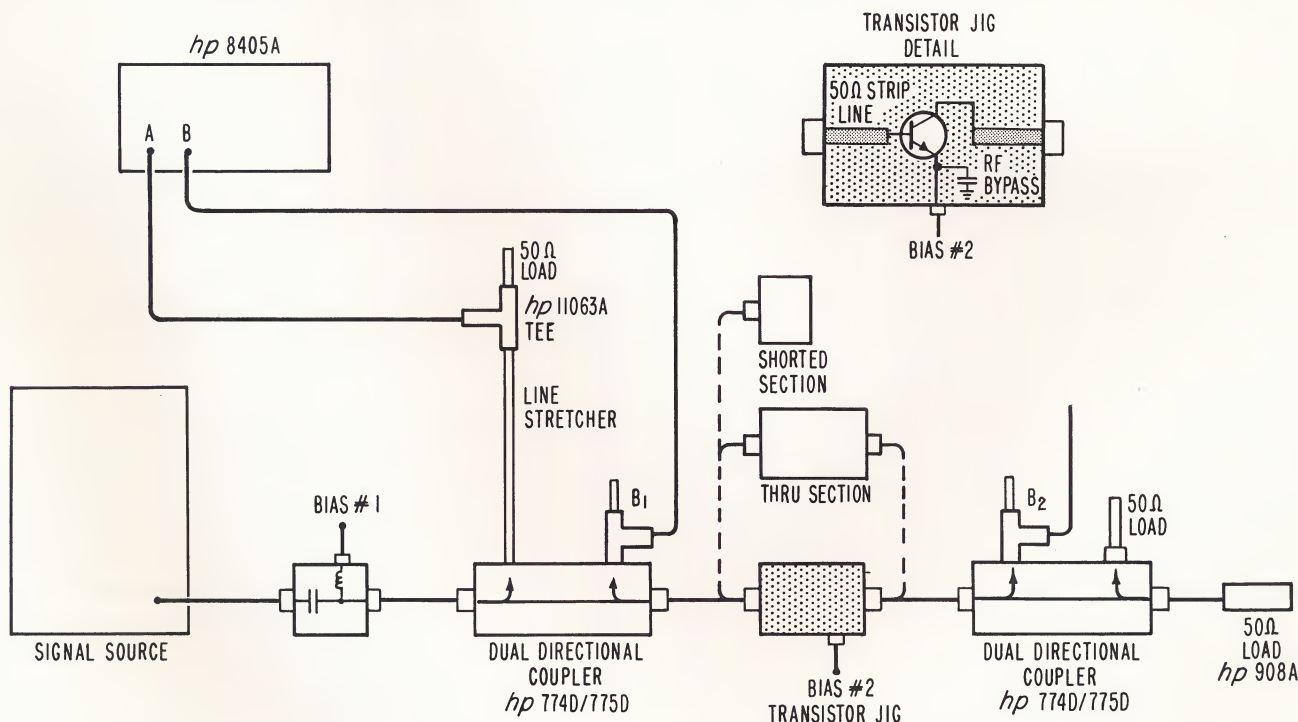


Figure 2. Block diagram of a typical s parameter measurement system. The through section and shorted section are used for calibration as described in the text. The transistor jig is shown in the forward direction as used in measuring s_{11} and s_{21} . The element for bias can be a commercial monitor tee such as a Microlab HW-02N (50 - 500 MHz) or HW-08N (350 - 1000 MHz). An appropriate line stretcher is a General Radio 874-LK20L.



Figure 3. Shorted section, through section and typical transistor jig for s parameter measurement setup. A transistor jig for the TO-18 package, common emitter, is available as specification number K02-8405A. A through section, K03-8405A, and shorted section, K04-8405A are also available. Prices on request.

similar to the transistor jig. Figure 3 illustrates a typical transistor jig, shorted section and through section.

To set the distance to A equal to the distance to B₁, place the shorted section in position as shown in Figure 2. Place probe B at B₁, apply an RF signal at a convenient level and adjust the line stretcher for 180°, the phase shift introduced by the short. Optimize the line stretcher position by checking phase over the frequency range of interest and centering the deviations from 180°. Place the through section in position and check for 0° phase shift with probe B in the B₂ position. If the shorted section is exactly half the length of the through section and the couplers are identical, no adjustment should be necessary.

Measurement Technique

To measure s_{11} insert the transistor in the jig, place the jig in the forward position and set bias #1 and bias #2 to the desired levels. Set the RF voltage at a convenient level (-30 dBm or less for most RF transistors) and measure the voltage at Channel A, the voltage at Channel B, and the phase angle ϕ . Then $s_{11} = \frac{B}{A} \angle \phi$. Ratio measurements are simplified if A is set

to unity -- 1 mV, 10 mV, etc. s_{22} is measured by reversing the jig and the dc bias connections. s_{21} can then be measured with probe B at B₂ and the jig in the forward position. Finally, by again reversing the jig and bias connections we can measure s_{12} . At each step, s would normally be measured at several frequencies.

Frequency Range of Measurement

Frequency range for the 774D is 215 to 450 MHz and for the 775D from 450 to 940 MHz, while the 8405A operates from 1 MHz to 1 GHz. Tests on several typical 774D's indicate that they are useable down to 100 MHz when the power in the main arm is -30 dBm. The nominal coupling coefficient is 20 dB in band and falls off at approximately 6 dB per octave below 215 MHz. Coupling tracking and phase tracking between auxiliary arms is quite good as the frequency is reduced so that the lower frequency limit is primarily due to the sensitivity of the 8405. The nominal maximum upper frequency is 1 GHz although useable measurements can be made to 1.1 GHz on most 8405A's.

Typical Values

s parameter measurements have been made on a number of popular RF transistors using the setup shown in Figure 2. The significance of s_{11} and s_{22} can easily be observed by plotting measured values on the Smith Chart. In Figure 4 s_{11} for a typical RF transistor is shown over the range from 100 MHz to 1200 MHz. Values of s_{11} , the input reflection coefficient, are entered by using the radial scale for magnitude and the polar coordinate scale for phase angle. Normalized input impedance of the device with output terminated in 50 ohms can then be read on the R and j X scales. Notice that at lower frequencies the overall input impedance is capacitive and that as the frequency is increased, inductive elements, primarily due to lead lengths, predominate. Figure 5 illustrates the behavior of s_{22} for the same transistor. In Section III we will discuss the use of Smith Chart plots of s_{11} and s_{22} in matching the input and output of the device for optimum circuit performance.

The s_{21} parameter indicates the basic forward gain of the device and is thus analogous to h_{21} . One useful way to plotting s is illustrated in Figure 6. From 100 to 1200 MHz the magnitude s_{21} for the measured transistor falls off at the expected rate of approximately 6 dB per octave. Measurements on a large number of transistors indicate that $|s_{fe}|$ is approximately equal to $|h_{fe}|$ (an exact conversion is given in Appendix I). Thus f_t , the frequency for which $|h_{fe}| = 1$, is approximately equal to f_{se} , the frequency for which $s_{fe} = 1$. When using h parameters h_{fe} is often measured at a relatively low frequency such as 100 MHz and f_t computed by extrapolation. Using the s parameter approach, the behavior of s_{fe} can easily be explored over a wide frequency range and f_{se} measured directly.

Still another useful way of exploring s_{21} is to observe its behavior at a given frequency as bias levels are

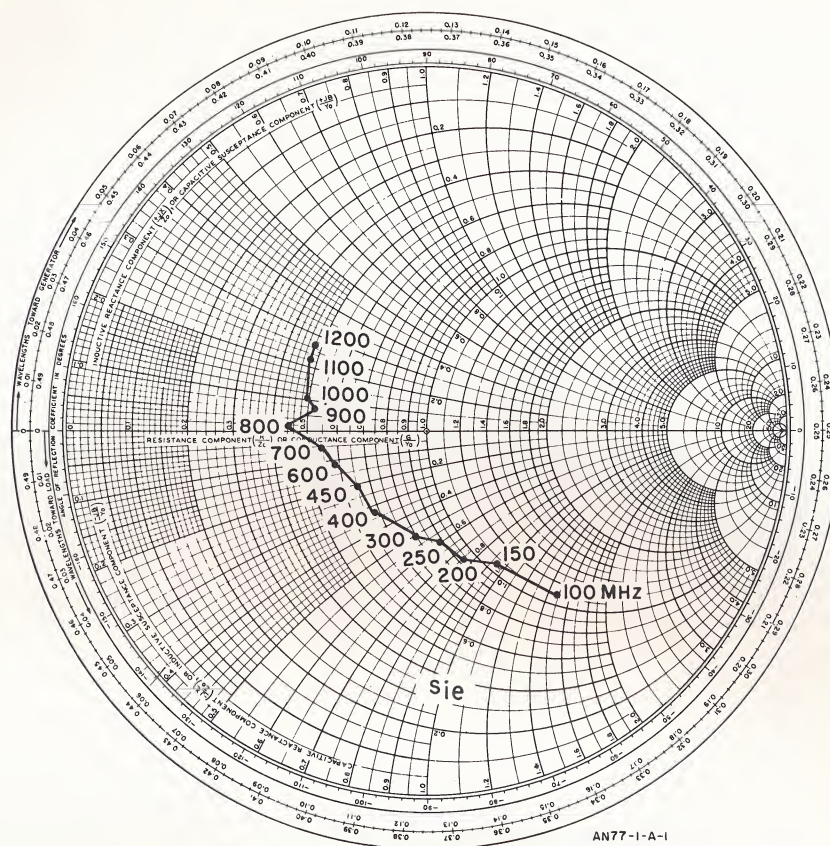


Figure 4. s_{ie} for a Fairchild 2N3563 Plotted from 100 MHz to 1200 MHz. $V_{ce} = 10$ V, $I_c = 5$ mA.

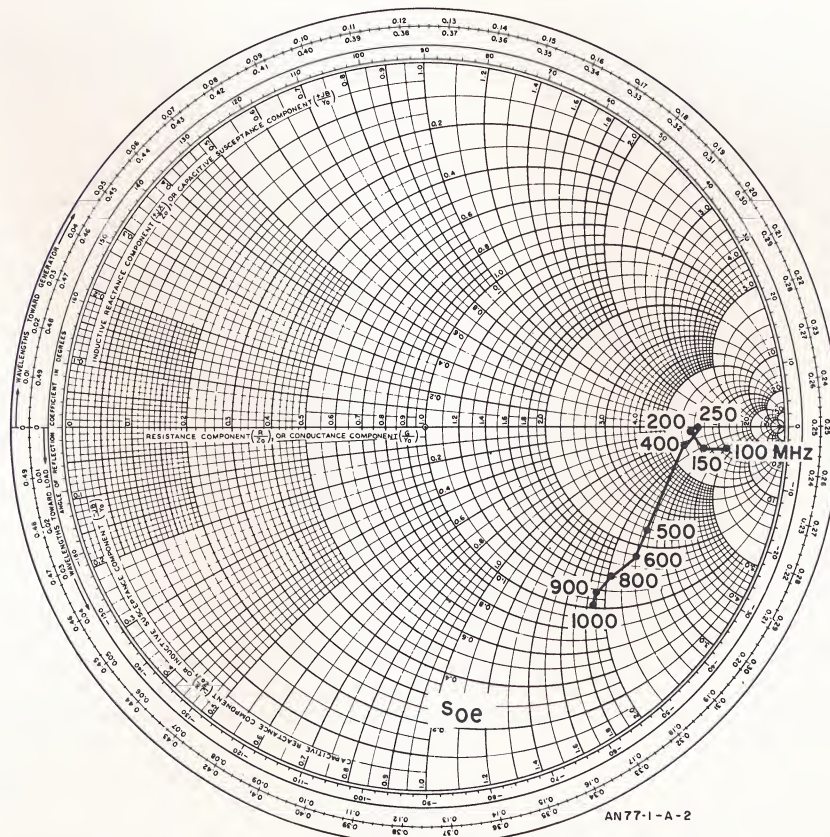


Figure 5. s_{oe} for a Fairchild 2N3563 Plotted from 100 MHz to 1000 MHz. $V_{ce} = 10$ V, $I_c = 5$ mA.

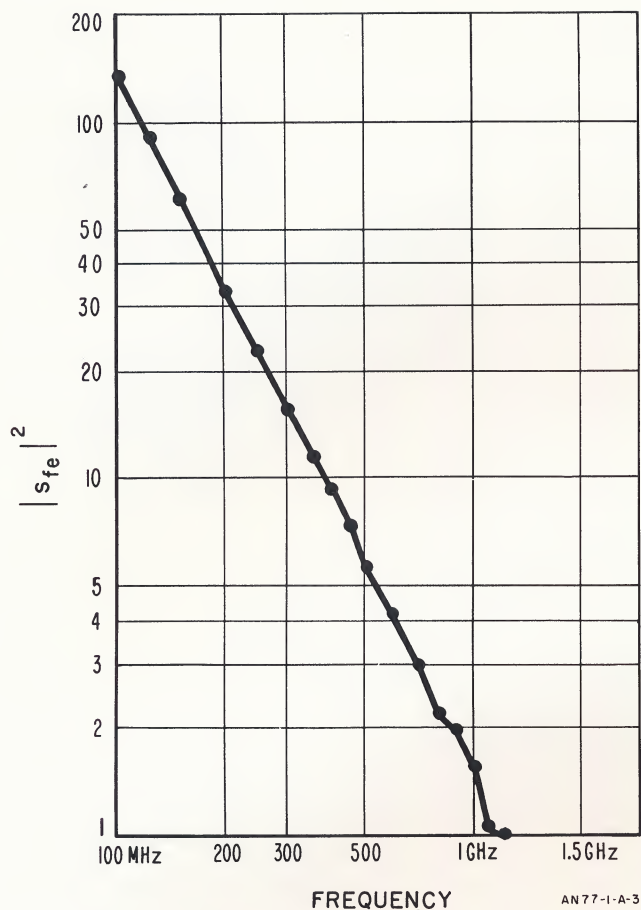


Figure 6. $|s_{fe}|^2$ vs frequency, RCA 2N857, grounded emitter. $V_{ce} = 6$ V, $I_c = 10$ mA, $f_t = 1000$ MHz.

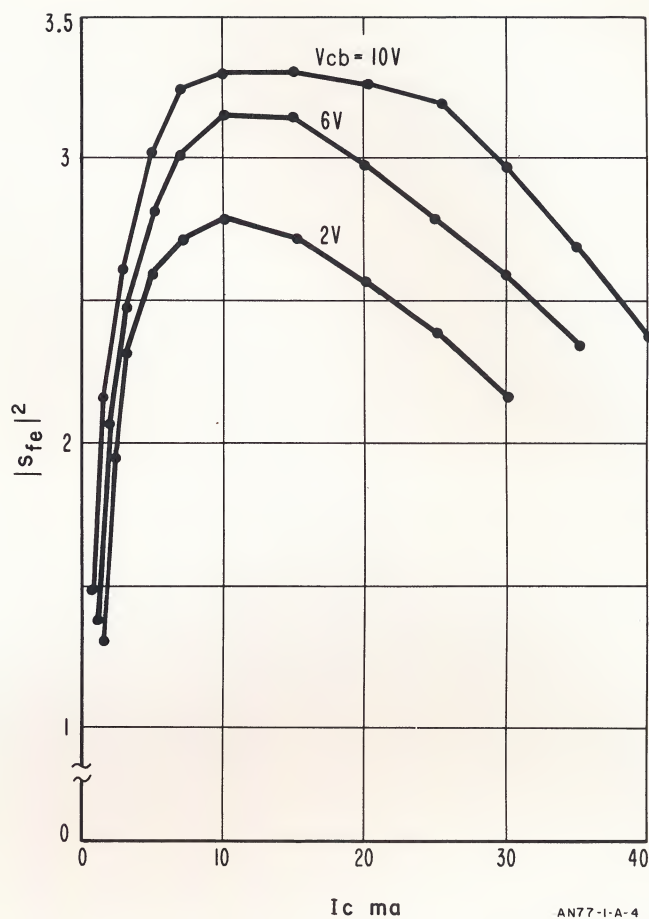


Figure 7. $|s_{fe}|^2$ vs I_c at 450 MHz, Fairchild 2N3463, grounded emitter.

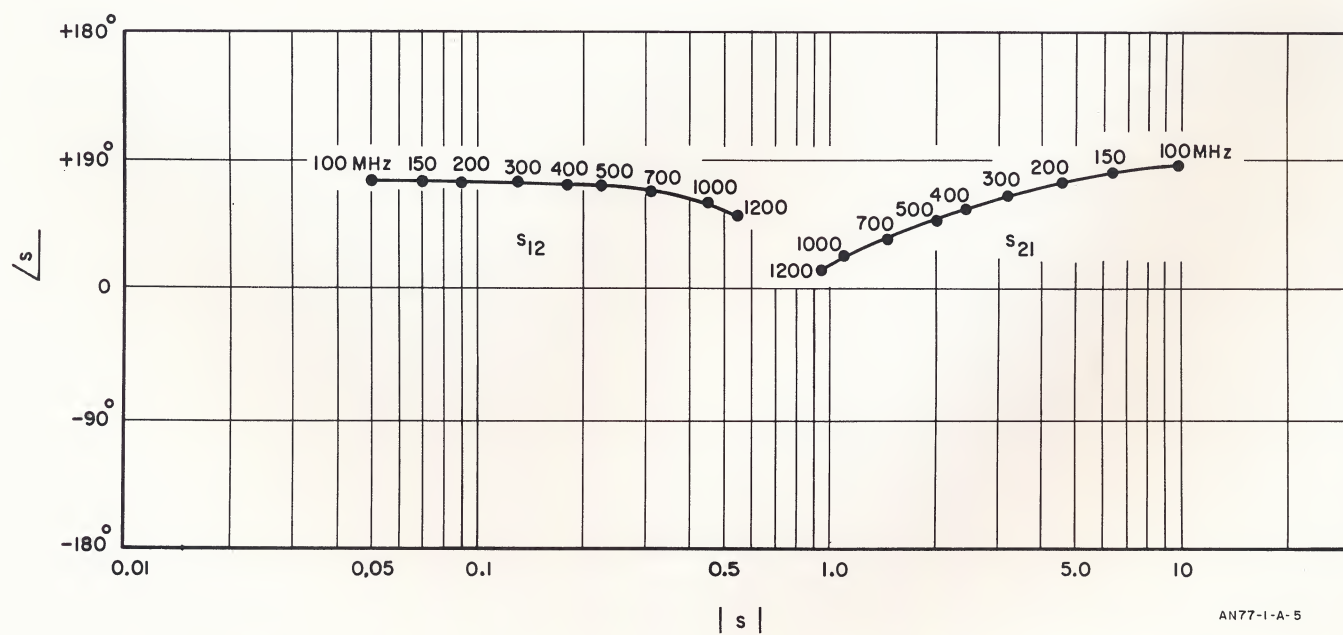


Figure 8. Magnitude and phase of s_{12} and s_{21} shown over the range from 100 MHz to 1200 MHz. Texas Instruments XS12, grounded emitter, $V_{ce} = 10$ V, $I_c = 50$ mA.

changed. Figure 7 illustrates the variation in $|s_{fe}|^2$ with collector current for various values of V_{cb} for a typical transistor operating at 450 MHz. This type of presentation can also be used to illustrate the variation of f_{se} with bias conditions.

The s_{12} parameter is proportional to the reverse power gain of the device, s_{12} typically increases with frequency while s_{21} decreases, so that $|s_{21} s_{12}|$ is approximately constant. For the transistor shown in Figure 7 the following results were obtained:

	$ s_{21} $	$ s_{12} $	$ s_{21} s_{12} $
450 MHz	1.7	0.10	0.17
1000 MHz	0.84	0.14	0.12

Figures 6 and 7 and the above table show magnitude information only for s_{12} and s_{21} . Figure 8 illustrates both magnitude and phase of s_{12} and s_{21} for another transistor from 100 MHz to 1000 MHz. This presentation would be particularly useful if the device is to be operated with 50 ohm source and load.

If $|s_{12}|$ is sufficiently small a simplified approach can be used to analyze the performance of the transistor with various loads and generator impedances. This technique is described in Section III.

Measurement Accuracy

The system shown in Figure 2 will give adequate accuracy for most circuit design applications. We will not attempt a complete error analysis in this Note but will simply point out the major sources of error and describe a few methods for achieving better accuracy.

Ideally, we would like the magnitude and phase of the signal delivered to probe A to be directly proportional to the incident voltage and the signal delivered to probe B proportional to the reflected or transmitted voltage. The accuracy of the measurement would then depend only on the 8405A. However, impedance mismatches between the elements in the system, coupler directivity, and variations in coupling coefficient tracking result in deviations from the ideal system. Mismatch errors can be minimized by selecting low SWR elements such as the 774D Directional Coupler and the 908A 50 ohm load and by careful design of the transistor jig. Coupler directivity should be as high as possible. Directivity of 40 dB will result in a maximum error of approximately 2% and 1°. Mismatch and directivity errors can be "tuned out" at a specific frequency if extreme precision is required. The measurement of $\left| \frac{B}{A} \right|$ is affected by the coupling factor of the auxiliary arm of the coupler. The coupling factor for the 774D and 775D is nominally 20 dB. Actual coupling factor for each auxiliary arm at 5 frequencies stamped on each 774D and 775D can differ from nominal by as much as ± 1 dB, resulting in a possible ± 1 dB error in measuring $\left| \frac{B}{A} \right|$ or $|s|$. This error can be corrected by using the data printed on the couplers or by plotting coupling curves using the 8405A and the shorted and through sections.

Instead of applying a correction to each measurement we have found the following procedure more convenient:

1. Set up the equipment as shown in Figure 2 with the shorted section in place.
2. Apply sufficient RF power to establish a reference level on Channel A such as 0 dBm.
3. Measure the power level at Channel B at 5 or 6 frequencies across the band keeping the power at A constant.

Typical data is shown in Table 1.

Table 1. Coupling Tracking - 775D

Frequency	Power at A	Power at B
215 MHz	0 dBm	+0.8 dBm
250	0	+0.6
300	0	+0.6
400	0	+0.3
450	0	+0.2

4. Adjust the gain of the B channel with gain control A18R11 to correct for the mean deviation measured. In the example given the gain would be reduced by 0.5 dB.
5. In the actual measurement the voltage at Channel A is then held at unity (i.e., 1 mV, 10 mV, etc.) and the voltage at B yields ratio directly. For the coupler shown in Table 1 the maximum error would be ± 0.3 dB. Measurements on a sample of 774D's and 775D's indicate coupler tracking is typically within ± 0.4 dB.

SECTION III

USE OF S PARAMETERS IN CIRCUIT DESIGN

Once the s parameters for a device have been measured, they can easily be used to determine overall gain with various source and load impedances. In addition, it is possible to establish source and load impedances which insure stability. Because s parameters can quickly be measured over a wide frequency range, broadband design is direct and simple. In this section we will explore the s parameter design approach for the important case where s_{12} is small. A derivation of the formulas used here and in Appendix II along with a more complete analysis of s parameter design is given in the following paper:

Bodway, George E., "Two Port Power Flow Analysis of Linear Active Circuits Using the Generalized Scattering Parameters." Copies available on request.

When s_{12} is small (see Appendix II for an evaluation of s_{12}) maximum gain is achieved when the input and output of the device are terminated in impedances which are the complex conjugate of s_{11} and s_{22} respectively. If r_1 is the generator reflection coefficient and r_2 is the load reflection coefficient, maximum gain occurs when $r_1 = s_{11}^*$ and $r_2 = s_{22}^*$. The gain, G_{\max} is given by the following expression:

$$(1) \quad G_{\max} = \frac{|s_{21}|^2}{(1 - |s_{11}|^2)(1 - |s_{22}|^2)}$$

This can be considered the product of three factors:

$$(2) \quad G_{\max} = |s_{21}|^2 \times \frac{1}{(1 - |s_{11}|^2)} \times \frac{1}{(1 - |s_{22}|^2)}$$

$$= G_0 \text{ (dB)} + G_1 \text{ (dB)} + G_2 \text{ (dB)}$$

Where G_0 is the forward power gain with the input and output terminated in 50 ohms, G_1 is the additional power gain (or loss) resulting from the impedance match between the device and the generator and G_2 is the additional power gain (or loss) due to the match between output and load. For any arbitrary source and load impedance the actual gain can be determined by graphical construction on the Smith Chart as follows:

The general expressions for G_1 and G_2 are:

$$(3) \quad G_1 = \frac{1 - |r_1|^2}{|1 - r_1 s_{11}|^2} \quad G_2 = \frac{1 - |r_2|^2}{|1 - r_2 s_{22}|^2}$$

Where r_1 and r_2 are the reflection coefficients of the load and generator with

$$(4) \quad r_1 = \frac{Z_g - Z_0}{Z_g + Z_0} \quad r_2 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

G_1 and G_2 are 0 dB with 50 ohm source and load. They are a maximum for conjugate source and load. Once s_{11} is measured, we can select a value for G_1 and solve for values of r_1 . The results plot as a circle of constant gain on the Smith Chart. The center of the circle lies on a line from the center of the Smith Chart passing through s_{11}^* . The radius of the circle, ρ_{01} , and the distance from the center of the circle to the center of the Smith Chart, r_{01} , are found by first defining a "normalized gain" g_1 where

$$(5) \quad g_1 = \frac{G_1}{G_{1\max}} = G_1 (1 - |s_{11}|^2)$$

then

$$(6) \quad r_{01} = \frac{g_1 |s_{11}|}{1 - |s_{11}|^2 (1 - g_1)}$$

$$\rho_{01} = \frac{(1 - g_1)^{1/2} (1 - |s_{11}|^2)}{1 - |s_{11}|^2 (1 - g_1)}$$

Example 1: Single Frequency Measurement at 500 MHz

$$s_{11} = 0.70 \angle -90^\circ \quad |s_{21}| = 2$$

$$s_{22} = 0.70 \angle -90^\circ \quad s_{12} = \text{negligible}$$

$$G_{\max} = \frac{|s_{21}|^2}{(1 - |s_{11}|^2)(1 - |s_{22}|^2)} = \frac{4}{1/2 \times 1/2} = 16 \text{ (12 dB)}$$

$$G_0 = 4 = 6 \text{ dB}$$

$$G_{1\max} = 2 = 3 \text{ dB}$$

$$G_{2\max} = 2 = 3 \text{ dB}$$

$$12 \text{ dB}$$

If we let $G_1 = 1 = 0 \text{ dB}$, $g_1 = 1/2$

$$r_{01} = \frac{1/2 \times 0.7}{1 - 1/2 \times 1/2} = 0.47$$

$$\rho_{01} = \frac{(1 - 1/2)^{1/2} (1 - 1/2)}{1 - 1/2 \times 1/2} = 0.47$$

We obtain the 0 dB gain circle which passes through the origin ($Z_g = 50\Omega$) as expected. By assuming other values for G_1 a family of constant gain circles is obtained as shown in Figure 8. Since, in our simple example we assumed $s_{11} = s_{22}$, the same family of gain circles applies to the device output. The values of r_1 and r_2 for maximum gain at 500 MHz are simply s_{11}^* and s_{22}^* or $0.70 \angle 90^\circ$. This indicates a generator and load impedance for conjugate match of $(0.35 + j0.95) \times 50 = 17.5 + j47.5\Omega$. A lossless matching network can now be inserted between a 50 ohm generator and the device and a similar network between the device and a 50 ohm load to achieve maximum gain. If for reasons such as optimum device noise performance or other circuit considerations a gain other than G_{\max} is desired, the constant gain circle giving possible matching impedances can be determined directly from Figure 9. The actual impedance of an experimental matching network can quickly be verified by inserting it in place of the transistor jig using the measuring setup shown in Figure 2.

Example 2: Broadband Design

A common objective for the circuit designer is to develop an amplifier having constant gain over a broad frequency range. Let us assume that we wish to operate the device from Example 1 at constant gain from 250 MHz to 500 MHz:

	At 250 MHz
$s_{11} = 0.87 \angle -50^\circ$	$ s_{21} = 2.5$
$s_{22} = 0.87 \angle -50^\circ$	$s_{12} = \text{negligible}$

We proceed by constructing a second set of constant gain circles and obtain the plot shown in Figure 10.

	At 500 MHz
$G_{\max} = G_0 \text{ dB} + G_{1\max} \text{ dB} + G_{2\max} \text{ dB}$	
$= 6 \text{ dB} + 3 \text{ dB} + 3 \text{ dB}$	$= 12 \text{ dB}$

	At 250 MHz
$G'_{\max} = 8 \text{ dB} + 6 \text{ dB} + 6 \text{ dB}$	$= 20 \text{ dB}$

Since we assumed $s_{11} = s_{22}$ at both frequencies, the sets of constant gain circles apply both to input and

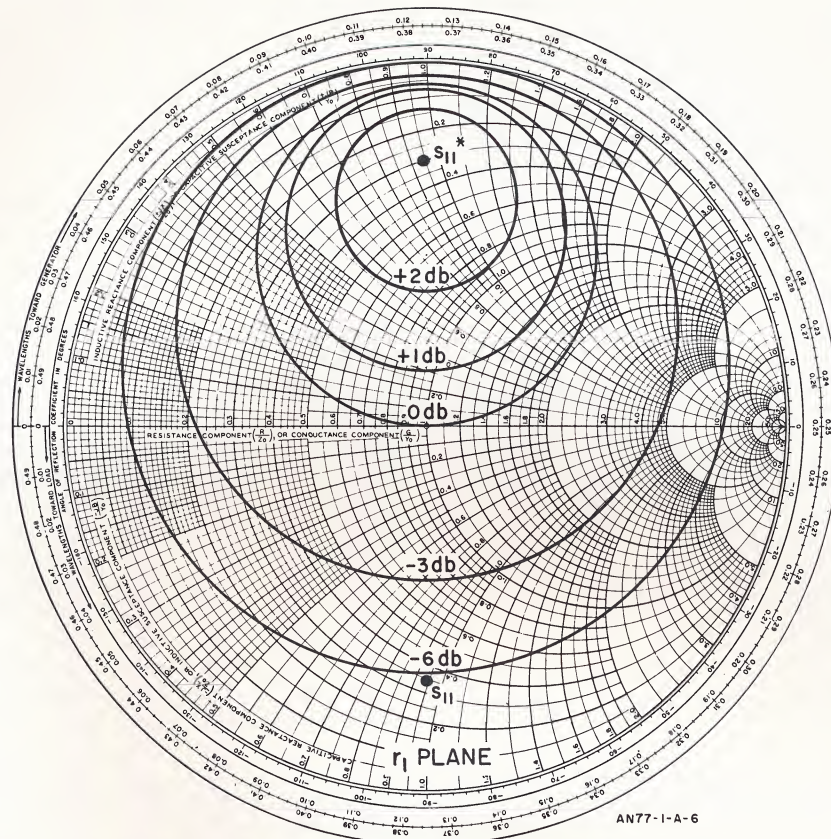


Figure 9. Circles of Constant Gain in the r_1 Plane for the Transistor in Example 1

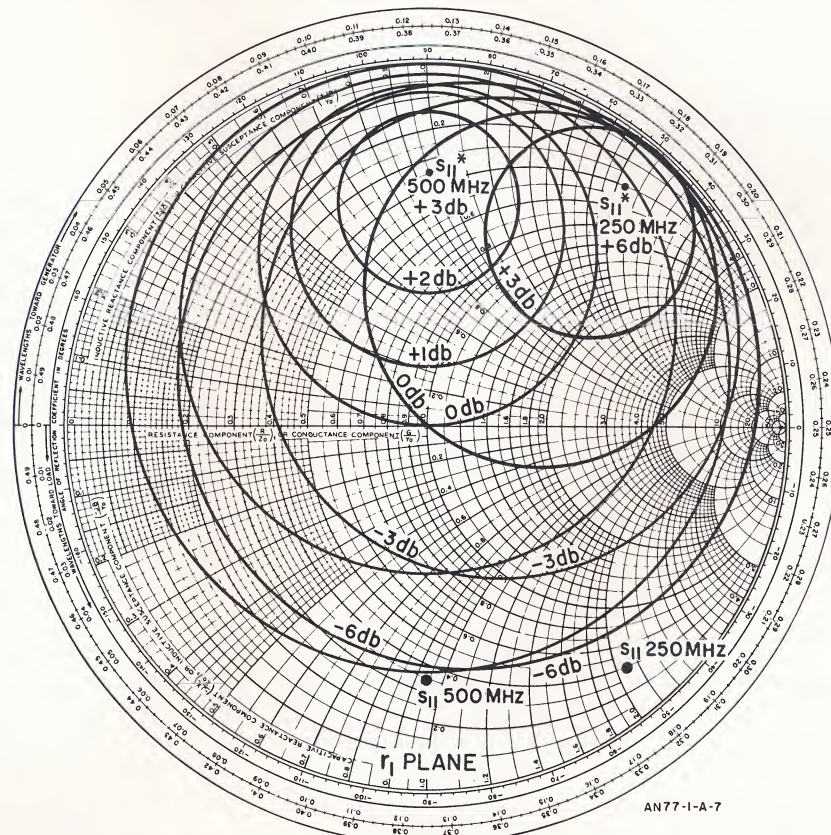


Figure 10. Two Sets of Constant Gain Circles in the r_2 Plane, One at 500 MHz and One at 250 MHz, Illustrating Broadband Design Described in Example 2

output of the device. We can now select a set of lossless matching elements for input and output which will conjugate match a 50 ohm source and load for maximum gain at 500 MHz and will lie on the 2 dB gain circles at 250 MHz so that the overall gain is 12 dB at both frequencies. Potential matching elements can be analyzed on the Smith Chart by following circles of constant impedance and constant conductance until a suitable combination is found. Checks would normally be made at one or two intermediate frequencies to verify broadband constant gain.

Example 3: Stability

If either s_{11} or s_{22} is greater than one, the device is potentially unstable and may oscillate under certain conditions of generator or load impedance. A very powerful aspect of the s parameter approach is that such values can normally be measured using the same setup as before. The Smith Chart can again be used to determine gain for various loads and generators and, in addition, can illustrate values that will result in instability. Again a simple example will be used to illustrate the technique:

$$s_{11} = 2.2 \angle 153^\circ = -2 + j1$$

Z_{11} , the input impedance with the output terminated in 50 ohms is then:

$$\frac{Z_{11}}{Z_0} = \frac{1 + s_{11}}{1 - s_{11}} = \frac{-1 + j1}{3 - j1} = -0.4 + j0.2$$

If we plot on the Smith Chart

$$\frac{1}{s_{11}} = \frac{1}{2.2 \angle -153^\circ} = 0.45 \angle 153^\circ$$

It turns out that Z_{11} can be read directly by interpreting the resistance coordinate as negative resistance and the reactive coordinate in the normal way.

$$\text{From Equation (3)} \quad G_1 = \frac{1 - |r_1|^2}{|1 - r_1 s_{11}|^2}$$

and we can see that for $r_1 = 1/s_{11}$ the gain goes to infinity, indicating unstable operation. Various values for G_1 can be selected and constant gain circles again plotted as in Examples 1 and 2 using the point $1/s_{11}$ as a reference. This is shown in Figure 11. Gain for different values of r_1 is indicated by the gain circles. The shaded portion of the chart indicates the unstable area. This can be seen intuitively by noting that whenever the real part of z_g is less than 0.4, the total resistance around the input loop is negative. We have explored the case for $|s_{11}| > 1$. The same approach can be applied to $|s_{22}| > 1$.

When $|s_{11}| > 1$ or $|s_{22}| > 1$ it turns out that the s parameters are much more convenient to measure than h ,

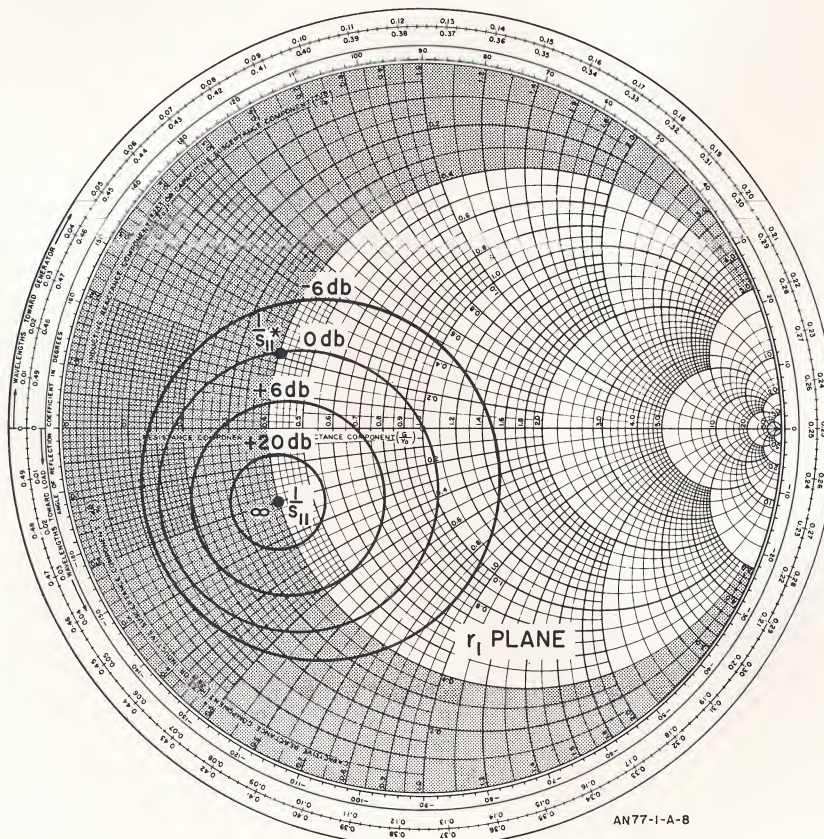


Figure 11. Set of constant gain circles in the r_1 plane for Example 3, $|s_{11}| > 1$. Shaded area of the Smith Chart indicates values for r_1 which result in unstable operation.

y, or z parameters. For the s parameter measurement, the input and output are terminated in 50 ohms over a large frequency range. Thus, as long as the negative real part of the series equivalent input or output impedance of the device does not exceed 50 ohms, the total loop resistance is positive and the device will not oscillate.

For h, y, and z measurements, the transistor is terminated with a short or open circuit. This is normally achieved at the plane of the device at a fixed frequency with a tuning stub. There is often some other frequency, then, at which the combination of the stub and the device will oscillate. While such oscillations can be suppressed, the measurement takes more time and corrections must be made to the readings.

CONCLUSIONS

As transistor circuits are used at increasingly higher frequencies, the need for a new method for small signal analysis has become critical. The use of s parameters appears to hold great promise for fulfilling this need since they are so convenient to apply to devices in transmission line applications. With the 8405A Vector Voltmeter s parameters are measured in a direct and simple manner. The s parameter approach has already been applied on several Hewlett-Packard R and D projects and has yielded considerable savings in design time. A number of transistor manufacturers are experimenting with s parameter measurements at the present time. Within the near future we expect to see increasing use of s parameters and expect them to appear on many RF device specification sheets.

APPENDIX I

$$s_{11} = \frac{(z_{11} - 1)(z_{22} + 1) - z_{12} z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12} z_{21}}$$

$$s_{12} = \frac{2z_{12}}{(z_{11} + 1)(z_{22} + 1) - z_{12} z_{21}}$$

$$s_{21} = \frac{2z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12} z_{21}}$$

$$s_{22} = \frac{(z_{11} + 1)(z_{22} - 1) - z_{12} z_{21}}{(z_{11} + 1)(z_{22} + 1) - z_{12} z_{21}}$$

$$s_{11} = \frac{(1 - y_{11})(1 + y_{22}) - y_{12} y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12} y_{21}}$$

$$s_{12} = \frac{-2y_{12}}{(1 + y_{11})(1 + y_{22}) - y_{12} y_{21}}$$

$$s_{21} = \frac{-2y_{21}}{(1 + y_{11})(1 + y_{22}) - y_{12} y_{21}}$$

$$s_{22} = \frac{(1 + y_{11})(1 - y_{22}) - y_{21} y_{12}}{(1 + y_{11})(1 + y_{22}) - y_{12} y_{21}}$$

$$s_{11} = \frac{(h_{11} - 1)(h_{22} + 1) - h_{12} h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12} h_{21}}$$

$$s_{12} = \frac{2h_{12}}{(h_{11} + 1)(h_{22} + 1) - h_{12} h_{21}}$$

$$s_{21} = \frac{-2h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12} h_{21}}$$

$$s_{22} = \frac{(1 + h_{11})(1 - h_{22}) + h_{12} h_{21}}{(h_{11} + 1)(h_{22} + 1) - h_{12} h_{21}}$$

$$z_{11} = \frac{(1 + s_{11})(1 - s_{22}) + s_{12} s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} s_{21}}$$

$$z_{12} = \frac{2s_{12}}{(1 - s_{11})(1 - s_{22}) - s_{12} s_{21}}$$

$$z_{21} = \frac{2s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} s_{21}}$$

$$z_{22} = \frac{(1 + s_{22})(1 - s_{11}) + s_{12} s_{21}}{(1 - s_{11})(1 - s_{22}) - s_{12} s_{21}}$$

$$y_{11} = \frac{(1 + s_{22})(1 - s_{11}) + s_{12} s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12} s_{21}}$$

$$y_{12} = \frac{-2s_{12}}{(1 + s_{11})(1 + s_{22}) - s_{12} s_{21}}$$

$$y_{21} = \frac{-2s_{21}}{(1 + s_{11})(1 + s_{22}) - s_{12} s_{21}}$$

$$y_{22} = \frac{(1 + s_{11})(1 - s_{22}) + s_{12} s_{21}}{(1 + s_{22})(1 + s_{11}) - s_{12} s_{21}}$$

$$h_{11} = \frac{(1 + s_{11})(1 + s_{22}) - s_{12} s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} s_{21}}$$

$$h_{12} = \frac{2s_{12}}{(1 - s_{11})(1 + s_{22}) + s_{12} s_{21}}$$

$$h_{21} = \frac{-2s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} s_{21}}$$

$$h_{22} = \frac{(1 - s_{22})(1 - s_{11}) - s_{12} s_{21}}{(1 - s_{11})(1 + s_{22}) + s_{12} s_{21}}$$

NOTE

h, y and z parameters listed above are all normalized to Z_0 . If h', y' and z' are the actual parameters, conversion is obtained as follows:

$$\begin{aligned} z_{11}' &= z_{11} \times Z_0 & y_{11}' &= \frac{y_{11}}{Z_0} & h_{11}' &= h_{11} \times Z_0 \\ z_{12}' &= z_{12} \times Z_0 & y_{12}' &= \frac{y_{12}}{Z_0} & h_{12}' &= h_{12} \\ z_{21}' &= z_{21} \times Z_0 & y_{21}' &= \frac{y_{21}}{Z_0} & h_{21}' &= h_{21} \\ z_{22}' &= z_{22} \times Z_0 & y_{22}' &= \frac{y_{22}}{Z_0} & h_{22}' &= \frac{h_{22}}{Z_0} \end{aligned}$$

APPENDIX II

The "smallness" of s_{12} can be evaluated by using the following expression which is developed in the paper by George Bodway referred to in the Introduction.

$$\frac{1}{|1 + |x|^2|} < \frac{|s_{21}'|^2_{\text{true}}}{|s_{21}'|^2_{\text{unilateral}}} < \frac{1}{|1 - |x|^2|}$$

where

$$|s_{21}'|^2_{\text{true}} = \text{True overall gain}$$

$$|s_{21}'|^2_{\text{unilateral}} = \text{"Unilateral" gain or gain calculated assuring } s_{12} = 0.$$

and

$$x = \frac{r_1 r_2 s_{12} s_{21}}{(1 - r_1 s_{11})(1 - r_2 s_{22})}$$

For the transistor shown in Figure 5 and 6, with conjugate match at input and output:

$$\begin{aligned} \text{At 250 MHz: } x &= 0.05 \\ \text{Unilateral Gain} &= \text{True gain} \pm .4 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{At 500 MHz: } x &= 0.07 \\ \text{Unilateral Gain} &= \text{True gain} \pm .6 \text{ dB} \end{aligned}$$